THE THEORY OF EFFICIENT GROWTH

by

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Theory of Efficient Growth

Abstract:

The main objective of this paper is to propose an analytical framework to examine the foundations of the theory of efficient growth. The theory of efficient growth is a newly developed theory based on the principles of the neoclassical framework. It argues that an economy grows efficiently under two conditions. First, that the public and the private sectors both perform independently from each other. Second, the sum of their independent performances reaches an equilibrium. This equilibrium determines the optimum point of economic growth, and this optimal point illustrates the efficiency of economic growth.

Keywords: economic growth, mathematical economics, economic theory, macroeconomics, business cycle, fiscal policy

1 – INTRODUCTION

In macroeconomics, the main subject of this branch is economic growth. Several theories have been developed to explain the conditions under which economic growth takes place and the parameters that determine its mechanism. The most memorable theories which marked this branch of economics are notably those of Robert Solow, who developed the Solow growth model, which emphasized the production function and the importance of technological progress;[1] Paul Romer, whose theory of economic growth focused on the endogenous factors of technological progress,[2] made a significant advancement in macroeconomics as well. Not all theories on this subject could be cited in this introduction, but the point is to stress the preponderance of economic growth and the crucial role that it plays in the business cycle.

The purpose of this paper is to make a contribution to the theory of economic growth by bringing a new perspective. What is this perspective? Besides Thomas R. Malthus who indirectly opened this field by making a country's population his central variable to explain the process of economic growth in his time, economic growth theorists generally all start with conventional variables such as capital and labor to then develop their theory. The perspective proposed in this paper starts with the public and the private sectors as the basic engine of economic growth rather than capital and labor.

The theory proposed in this paper argues that the expansion of growth reaches an optimal point, and this optimal point determines the efficiency of economic growth. In other words, this theory maintains that economic growth is efficient under two fundamental conditions: (1) that the

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[1] Solow, Robert. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics.* Vol. 70, No. 1. (1956). pp. 65-94

[2] Romer, Paul. "Endogenous Technological Change." *The Journal of Political Economy.* Vol. 98. No.5. Part 2: The Problem of Development: A Conference of the Institute for the Study of Free Enterprise Systems. pp. 71-102.

performances of the public and the private sectors are independent of one another, and (2) that the sum of the performances of both sectors reaches an equilibrium, and this equilibrium determines the efficiency at which the economy performs overall.

This paper uses rigorous analytical methods based on the principles of neoclassical economics to methodically analyze the mechanisms that determine the theory of efficient growth. Lastly, it is important to clarify that the model used to illustrate this theory is not exhaustive. This theory will surely be refined by other economists who would want to expand on the framework that was developed.

2 – THEORETICAL FRAMEWORK

2.1. CONDITIONALITY OF SECTOR'S PERFORMANCE

a) The Private Sector

The private sector is determined by two essential factors: private consumption, i.e., the income obtained by the consumer after taxes (disposable income) as well as his propensity to consume, and private investment, i.e. the savings that the consumer uses to invest in the economy. It is important to state that the private sector is the sector that creates wealth since it is the sector wherein all economic activities mainly take place. Production and distribution of goods and services are generally concentrated in the private sector. Thus, since the private sector is based on private consumption and private investment, this could be written as the following function:

$$PV = f(C, I)$$
$$PV = C + I$$

Where (PV) represents the private sector, (C) represents consumption and (I) represents investment. During periods of economic expansion, consumers tend to spend more because they have confidence in the economy. Moreover, periods of economic expansion increase consumers' income and create employment because the sector provides value to the economy. When the economy however begins to fall, consumers then begin to save more as their confidence in the private sector substantially decreases. But this shows the opportunity to create value by investing more in the economy. Hence, this could be written as mathematically as:

$$Y = C > I$$
$$Y = C < I$$

In this small equation, (Y) represents the economy while (C) and (I) represent consumption and investment, respectively. Evidently, the problem is that this change in consumer behavior does not determine when the private sector is always the most beneficial to the consumer. If during times of expansion the consumer prioritizes consumption over savings and investing, and during times of recession, the consumer prioritizes the opposite, which means that the level of money used to conduct economic activities in both features is not Pareto efficient. Consequently, the real question to ask ourselves is to know at what point in time the private sector is considered efficient? Logically the private sector would be considered efficient at its optimal point if the level of consumption is equal to the level of investment. This could be translated as:

$$C = I$$

To show, however, the efficiency of the performance of the private sector as a whole, let us rewrite the equation in a more comprehensive form:

$$\sum_{t=1}^{k} PV_t(C_t + I_t) = 0 \qquad (1)$$

Where (k) represents the consumer, (t) represents a given period in time, and PV represents the private sector. It is noteworthy to emphasize that the efficiency of the private sector's performances is a dynamic movement of the private sector with respect to time. Thus equation (1) will then be transformed into a linear differential equation:

$$\frac{dPV}{dt} + (C + I)PV = 0 \qquad (2)$$

Since we have a linear differential equation to solve, let us transform the variable $(C + I)$ in equation (2) into a single variable X. Thus, this will give us equation 3:

$$\frac{dPV}{dt} + XPV = 0 \quad (3)$$

Let us find the integrating factor to start solving this linear differential equation[3]. The formula of

the integrating factor is:

$$I(x) = e^{\int P(x)dx}$$

Hence, we have:

$$I(t) = e^{\int Xdt}$$

$$I(t) = e^{\frac{1}{2}X^2}$$

Now that we have determined the integrating factor, let us find the general solution. The formula

of the general solution for a linear differential equation is:

$$Y = \frac{1}{I(x)}\left(\int I(x)Q(x)dx + c\right)$$

Let us then replace this formula with our variables:

$$PV = \frac{1}{e^{\frac{1}{2}X^2}}\left[\int e^{\frac{1}{2}X^2} 0 \, dt + c\right]$$

$$PV = \frac{1}{e^{\frac{1}{2}X^2}}\left[\int 0 \, dt + c\right]$$

$$PV = \frac{1}{e^{\frac{1}{2}X^2}}[c] = \frac{c}{e^{\frac{1}{2}X^2}}$$

Therefore:

$$PV = c \cdot e^{-\frac{1}{2}X^2} \quad (3)$$

The result of equation (3) shows that the performance of the private sector is efficient when

the economy is about to enter a downturn. This makes sense because it is at the period that

consumers begin to adjust their behavior toward saving and investing. It is remotely impossible

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[3] Equation (3), $P(t) = X$, $Q(t) = 0$.

for the private sector to perform efficiently during times of aggressive economic expansion because consumers are driven by spending and taking higher risks since interest rates are generally lowered during that period.

It is imperative to reiterate that the word 'efficient' in the context of this theory aligns with the concept of Pareto efficiency. The performance of the private sector is considered efficient if consumption criteria can be better off without making saving and investing criteria worse-off. If the economy expands indefinitely, then consumers will never save an important portion of their income and invest that portion back in the economy. Private investment is what makes the economy grow, and the economy cannot grow efficiently if private consumption and private investment are not at the same level.

b) The Public Sector

Unlike the private sector where most economic activities take place, the public sector does not create wealth. The relationship of the public sector to the economy is the application of economic regulations to the market. Economic regulations do not directly create value for the taxpayer in the sense that he benefits from it. In the private sector, the taxpayer, who is, therefore, the consumer, benefits directly from the value that the sector created through the products and services consumed. Hence, in the private sector, the taxpayer consumes and invests to maximize his welfare. This does not mean that the public sector produces no value at all. On the contrary, it does. But the value produced by the public sector is simply different from that of the private sector. The value produced by the public sector allows economic activities to operate within a legal framework where consumers and producers are all protected.

The public sector contributes to economic growth by investing in infrastructures, intelligence services, and national defense. Public sector investments in the economy are based on two factors: taxes, and the government budget. These two factors could be represented by the following function:

$$PB = f(T_t, GB)$$
$$PB = T_t + GB$$

Where (PB) represents the public sector, (T_t) represents absolute tax at a given period in time, and (GB) represents government budget. Government budget consists of government spending and government revenues, which could be denoted as (G_S) and (G_R), respectively. Hence, we can rewrite the function as:

$$PB = T_t + G_s + G_R$$

The performance of the public sector fluctuates according to the government budget. When public expenditures exceed public revenues, which come from taxes, it then creates a budget deficit. And a budget surplus occurs when public revenues exceed public spending. This relationship could be characterized by the following system of equations where (BD) is denoted as budget deficit and (BS) is denoted as budget surplus:

$$BD = G_S > G_R$$
$$BS = G_S < G_R$$

Budget deficit generally reduces investments, which leads to a bad economy. Excessive government spending penalizes the economy as a whole because it creates a debt that must be paid off. In doing so, the government fails to invest in infrastructures and public services to contribute

to economic growth. Moreover, budget deficit forces government to increase taxes, reduce public services, and increase the price of commodities which leads to inflation and consequently lowers the standard of living of the taxpayer.[4] Although some economists see budget deficit as an asset for short-term growth, most economists agree that budget deficit has a more negative impact on the economy than a positive one. Budget surplus, on the other hand, is perceived by economists as an added value to the economy. It is even advised to have a budget surplus rather than a budget deficit.

One of the greatest advantages of budget surplus is that the surplus is used to reduce the existing debt.[5] And the reduction of the debt enables further government investments.[6] Moreover, budget surplus enables fiscal flexibility and substantially reduces inflation, which in turn, favors economic growth.[7] However, budget surplus is also a double edge sword. An excess of budget surplus can indeed negatively impact the economy as well, which will make its growth less efficient. One of the major drawbacks of budget surplus is that it brings the economy into a deflationary state. When government operates a budget surplus, it is removing money from circulation in the wider economy. With less money circulating, it can create a deflationary effect. Deflation is not inherently problematic, but an excess of deflation can lead to adversarial consequences for economic growth. Excessive deflation discourages consumer spending, and the continuous fall of commodity prices cheapens the value of goods and services.[8] Saving and investing triggers economic growth but saving and investing without spending slows the economy

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[4] Huntley, Jonathan. "The Long-Run Effects of Federal Budget Deficits on National Saving and Private Domestic Investment." *Working Paper Series.* Working Paper 2014-02. (2014). Congressional Budget Office. pp. 1-9
[5] Auerbach, Alan J.; Gale, William G. "Perspective on Budget Surplus." *National Tax Journal.* Vol. 53, No. 3. (2000). pp. 459-472
[6] Ibid. p. 460
[7] Ibid. p. 461
[8] Alesina, Alberto. "The Political Economy of the Budget Surplus in the United States." *Journal of Economic Perspectives.* Volume 14, Number 3. (2000). pp. 3-19

because it generates unemployment.[9] If consumers do not spend, no profit could be made, and no workers can be paid if firms have no income.

As we have seen, the performance of the public sector is neither efficient under a budget deficit nor under a budget surplus. The subsequent question, therefore, is to know under what conditions the performance of the public sector is considered efficient? The performance of the public sector is considered efficient if the budget is balanced, i.e. public revenues and expenditures are more or less at the same level. This could be written mathematically as the following where (BB) represents the balanced budget:

$$BB = (G_S = G_R) \Longleftrightarrow G_S \leq G_R$$
$$BB = G_s \leq G_R$$

Realistically, government spending and government revenues can never be exactly at the same level. One will always surpass the over. As a general rule of thumb, a balanced budget is ensconced when government revenues are equal or greater than government spending to the slightest degree. Thus, the comprehensive form of the public sector performance at its efficient point could be then written equation 4:

$$\sum_{t=1}^{k} PB_t(T_t + BB_t) = 0 \qquad (4)$$

Where (k) represents the taxpayer, (t) represents a given period in time, (PB_t) represents the public sector at a given period in time, (T_t) represents absolute tax at a given time period, and (BB_t) which represents the balanced budget at a given point in time. Like in the private sector, the performance

[9] Ibid. p. 5

of the public sector is based on how the public sector operates with respect to time. This we will

then have the following linear differential equation:

$$\frac{dPB}{dt} + BBPB = T \quad (5)$$

Equation (5) looks similar to equation (2). The only difference, yet substantial is that the linearity

is equal to a variable on the other side of the equation. Since the dynamic of the performance of

the public sector is mainly based on the government budget, taxes are only affected if government

budget changes. (BB) could be replaced by a single variable X like in equation (3), which will

allow us to solve this differential equation more easily. Thus, we will have:

$$\frac{dPB}{dt} + XPB = T \quad (6)$$

Let us solve equation[10] (6) by applying the conventional rules and properties of the linear

differential equation. First, let us find its integrating factor:

$$I(t) = e^{\int P(t)dt}$$
$$I(t) = e^{\int Xdt}$$
$$I(t) = e^{\frac{1}{2}X^2}$$

Now that we determined the integrating factor, let us find the general solution of the equation.

Since the formula of the general solution of the linear differential equation was written as part

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[10] For Equation (6), $P(t) = X$ and $Q(t) = T$

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solving equation (3), there is no need to rewrite again that same formula. Let us directly apply to equation (6). This will then give us:

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\int e^{\frac{1}{2}X^2}.T \, dt + c\right]$$

After multiplying the integrating factor by Q (*t*), we then have:

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\int Te^{\frac{1}{2}X^2} \, dt + c\right]$$

Let us now focus on solving the integral that is within the bracket. Hence, let us use integral by substitution:

$$\begin{cases} u = \frac{1}{2}X^2 \\ du = Xdt \\ \frac{du}{X} = dt \end{cases}$$

Hence, let us substitute the variables of this integral in bracket. We will then have:

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\int Te^u\left(\frac{du}{X}\right) + c\right]$$

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\frac{1}{X}\int Te^u du + c\right]$$

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\frac{Te^u}{X} + c\right]$$

$$PB = \frac{1}{e^{\frac{1}{2}X^2}}\left[\frac{Te^{\frac{1}{2}X^2}}{X} + c\right]$$

Now that the integral has been solved through the method of substitution, let us proceed to the last steps to find the general solution:

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$$PB = \left[\left(\frac{1}{e^{\frac{1}{2}X^2}} \times \frac{Te^{\frac{1}{2}X^2}}{X}\right) + \left(\frac{1}{e^{\frac{1}{2}X^2}} \times c\right)\right]$$

$$PB = \left[\left(\frac{Te^{\frac{1}{2}X^2}}{Xe^{\frac{1}{2}X^2}}\right) + \left(\frac{c}{e^{\frac{1}{2}X^2}}\right)\right]$$

Consequently, the general solution of equation 6 is:

$$PB = \frac{T}{X} + c.\,e^{-\frac{1}{2}X^2} \qquad (6)$$

The general solution of equation (6) could be re-written as the following:

$$PB = \frac{T}{BB} + c.\,e^{-\frac{1}{2}X^2} \qquad (6)$$

The general solution of equation (6) shows that the performance of the public sector is Pareto efficient when the balanced budget can be better off without making taxes worst-off. By making taxes worse-off, we mean an increase in taxes. The performance of the public sector is then efficient when the balanced budget decreases toward the mean. Let us not forget that the balanced budget is realistically never equal to zero ($G_S = G_R$). One will always outweigh the other. Since government revenues are meant to be equal or greater than government spending, the efficient point of the performance of the public sector implies that the balanced budget must be closer to zero than marginally above it. If the balanced budget is closer to the mean (zero), then taxes will be also maintained closer to the mean.

2.2. EQUILIBRIUM OF EFFICIENT GROWTH

Economic growth is considered efficient when the performances of the public and private sectors reach an equilibrium. In the first part of our analysis, we rigorously demonstrated the process through which the performance of each sector reaches an equilibrium in their respective sphere. In this part, we are going to demonstrate how both, the public and private sectors, reach an equilibrium that illustrates the efficiency growth of the economy. The efficient growth of the economy could be represented by equation 7:

$$\sum_{t=1}^{k} GDP_t(PB_t + PV_t) = 0 \quad (7)$$

Where (k) represents the population, (t) represents a given period in time, (GDP) represents the economy, (PB_t), and (PV_t) represent the performance of the public sector and private at their respective efficient point in their respective sphere at a given period in time. Equation (7) is the sum of equation (6) and equation (3). Let us rewrite equation (7) in a more comprehensive form. This will then give us equation (8):

$$\sum_{t=1}^{k} GDP_t \left[\left(\frac{T}{X} + c. e^{-\frac{1}{2}X^2} \right) + \left(c. e^{-\frac{1}{2}X^2} \right) \right] = 0 \quad (8)$$

If we factorize $\left(c. e^{-\frac{1}{2}X^2} \right)$ then we will have:

$$\sum_{t=1}^{k} GDP_t \left[c. e^{-\frac{1}{2}X^2} \left(1 + \left[\frac{T}{X} + 1 \right] \right) \right] = 0 \quad (8)$$

The efficient growth of an economy is the sum of two partial equilibriums. This will then lead us to equation (9), which is a partial differential equation to solve in order to find the optimal point of growth, which will represent the efficient point of economic growth. Equation (9) is written as:

$$\frac{\partial \text{GDP}}{\partial \text{PV}} + \frac{\partial \text{GDP}}{\partial \text{PB}} = 0 \quad (9)$$

We have a first-order partial differential equation to solve. Before solving it, let us substitute our main variables with common variables to facilitate the solution process. Let us then replace GDP, PV, and PB with common variables:

$$\begin{cases} \text{GDP} = \text{U} \\ \text{PV} = x \\ \text{PB} = y \end{cases}$$

Thus, equation (9) will become equation (10):

$$\frac{\partial \text{U}}{\partial x} + \frac{\partial \text{U}}{\partial y} = 0 \quad (10)$$

Let us solve equation (10) by applying the rules of first-order partial differential equations. Let us commence by determining the partial derivatives of our two independent variables:

$$X(x)Y(y) = 0$$

$$\begin{bmatrix} \dfrac{\partial \text{U}}{\partial x} = \text{dX. Y} \\ \dfrac{\partial \text{U}}{\partial y} = \text{X. dY} \end{bmatrix} \Rightarrow \text{dX. Y} + \text{X. dY} = 0$$

Now that we determined the partial derivatives of each independent variable, let us separate these variables:

$$\frac{\text{dX}}{\text{X}} = -\frac{\text{dY}}{\text{Y}} = \text{k (constant)}$$

The separation of variables leads us to two ordinary differential equations that need to be solved:

$$a) \frac{dX}{Xdx} = k$$

And:

$$b) -\frac{dY}{Ydy} = k$$

Let us then solve the first ordinary differential equation:

$$a) \frac{dX}{Xdx} = k$$

$$\frac{dX}{X} = k\,dx \Rightarrow \frac{1}{X} = k\,dx$$

Let us integrate both sides:

$$\int \frac{1}{X} = \int k\,dx$$

$$\ln X = kx + c_1$$

$$X = e^{kx+c_1}$$

$$X = c_1.e^{kx} \Longleftrightarrow PV = c_1.e^{kx}$$

Let us now solve the second ordinary differential equation:

$$b) -\frac{dY}{Ydy} = k \Rightarrow \frac{dY}{Ydy} = -k$$

$$\frac{dY}{Y} = -k\,dy \Rightarrow \frac{1}{Y} = -k\,dy$$

Let us integrate both sides:

$$\int \frac{1}{Y} = \int -k\,dy$$

$$\ln Y = -ky + c_2$$

$$Y = e^{-ky+c_2}$$

$$Y = c_2.e^{-ky} \Longleftrightarrow PB = c_2.e^{-ky}$$

Now that we solved both ordinary differential equations, let us rewrite equation (10) in its most complete form:

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = c_1 . e^{kx} + c_2 . e^{-ky} \quad (10)$$

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = c_1 . c_2 + e^{kx} . e^{-ky}$$

Let us replace $(c_1 . c_2)$ by (A) and factorize $(e^{kx} . e^{-ky})$. Then, we will have:

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = A e^{k(x-y)}$$

Therefore, the economy is considered to be growing efficiently when:

$$GDP = A e^{k(x-y)} \quad (10)$$

2.3. EXPLANATORY AND PREDICTIVE POWERS OF THE EFFICIENT GROWTH THEORY
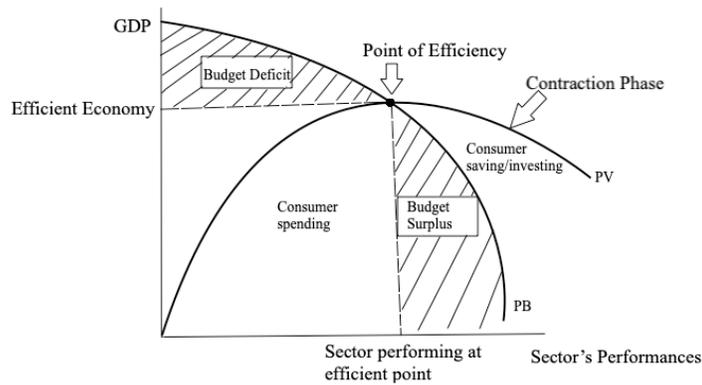
Efficient Growth Model

Figure 1

a) Explanatory Power of the Model

The efficient growth model enables us to comprehend how the performances of the private sector, as well as that of the public sector, intertwine in the economy. Both, the private and the public

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sectors, have an exponential growth shape. The performances of the private sector have a positive exponential growth whereas those of the public sector have an exponential decay.

The point of efficiency suggests that the economy operates within a balanced budget from the public sector. As the economy operates within a balanced budget, the level of taxes remains relatively stable. Since the level of taxes is stable and we assume that wages increase periodically, taxpayers then get to retain a higher portion of their income, which in turn increases their purchasing power. With more money in their pockets, taxpayers can spend and invest more as well in the economy. We can see that the level of spending and investing in the economy from the consumer depends on how the government budget is used.

If the budget is not balanced, then taxpayers will either spend more or save more. In either situation, economic growth is not at its optimal point. For example, if the public sector spends more, then consumers in the private sector will have the save more because the accumulated debt that needs to be paid off due to increasing public spending will force government to raise taxes, which will subsequently decrease the purchasing power of consumers. If, however, the public sector is extremely frugal (although recommended, it could potentially have negative consequences), this then will lead to a decrease in taxes, which means higher consumer spending in the private sector and higher spending will create an overheated economy that will eventually lead to an economic downturn. In short, the economy is at its efficient point when the public sector balances the budget, and the private sector enters a phase of economic contraction.

b) Predictive Power of the Model

The efficient growth model predicts several outcomes. The first outcome is based on the relationship between private consumption and GDP. This relationship should statistically result in a positive relationship. The second outcome is based on the relationship between private

investment and GDP. The empirical result of this relationship should provide a moderately weak negative relationship during phases of economic contraction. During periods of expansions though, private investment will be positively correlated with GDP. The third outcome of the model's prediction is to obtain a positive relationship between the balanced budget and GDP.

3 – CONCLUSION

The purpose of this paper was to establish the theoretical foundations of the theory of efficient growth. The foundations of our theory assume that in order to understand the mechanisms of economic growth, it is important to assess the optimal point of that growth because it is that point that whether determines if the economy is performing efficiently or not.

The theory of efficient economic growth leads economists to think differently on this subject. This theory makes it possible to precisely determine the instances whereby economic growth changes dynamics. And to be able to precisely determine these dynamic changes, it is crucial to assess the optimal point wherein the economy performs most efficiently.

Lastly, we know that this theory is not flawless. It surely contains some drawbacks that will need to be eventually assessed. As the famous British statistician, George Box, once famously said: "all models are wrong, but some are useful." The model that we developed surely contains some limitations. But the most important is to make this theory useful by providing empirical analysis that will either back or falsify its major assumptions. If the results back the assumptions of this theory, we could then consider this theory to be empirically valid. If the data, however, contradict our assumptions, then we will work on refining our model and improving its predictive power. Scientific research is a perpetual collaboration, and we hope at least that the theoretical

foundations of our hypothesis will prompt economists to continue working on the theory of efficient growth and bring a new perspective to it.

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3. Equation (3), P (t) = X, Q (t) = 0.

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9. Ibid. p. 5

10. For Equation (6), P(t) = X and Q(t) = T